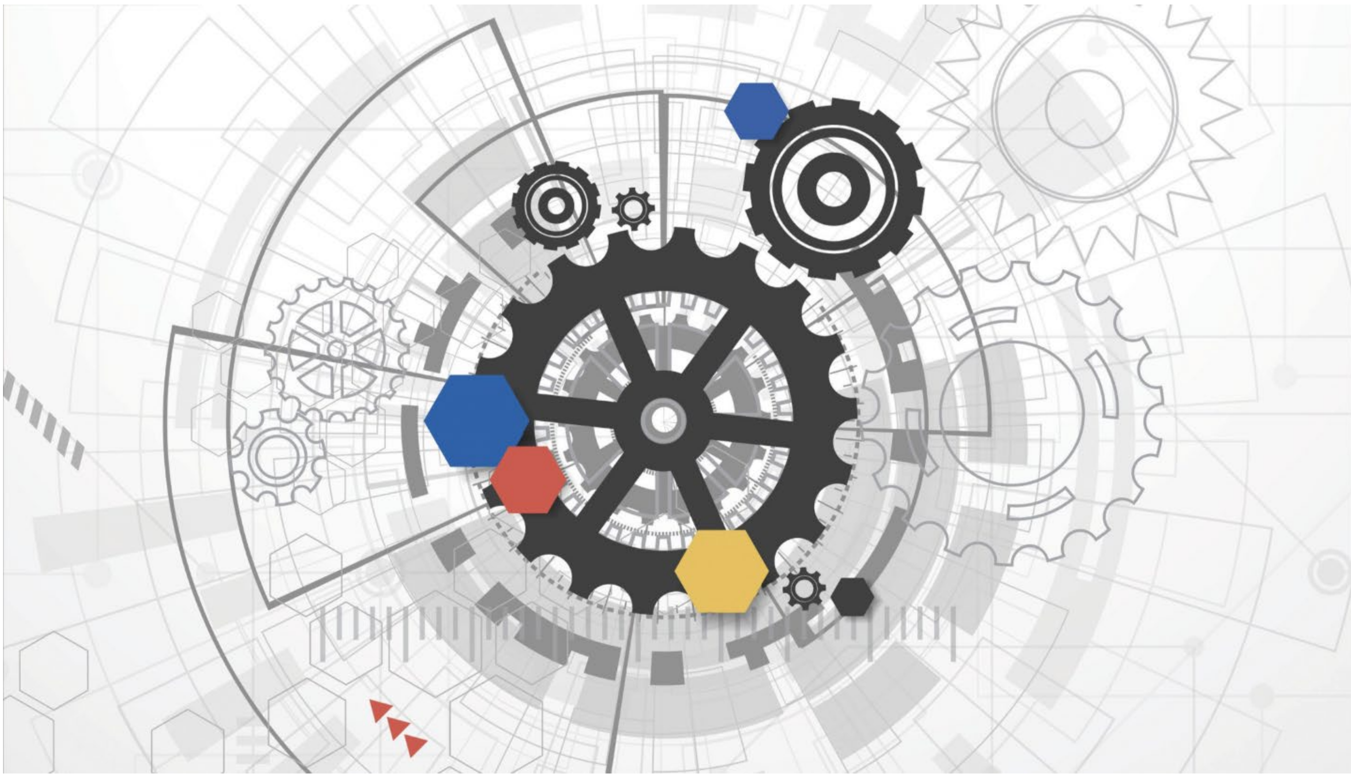

Mechanical Science

DYNAMICS

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Dynamics

Motion

ACCELERATION

This refers to the motion of a body whose velocity is not uniform. Suppose the velocity of a body goes on changing at a uniform rate, then the rate of change of velocity is called *acceleration.* This is denoted by "f" or "a".

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time taken}}$$

EQUATION OF MOTION

1. $v = at + u$
2. $s = \frac{at^2}{2} + ut$
3. $v^2 = 2as + u^2$

These equations have been obtained for the case of a body moving with *positive* uniform acceleration. If the body be moving with *negative* uniform acceleration i.e. with uniform retardation of f units per sec^2 the following relations will hold good.

1. $v = u - at$
2. $s = ut - \frac{1}{2}at^2$
3. $v^2 = u^2 - 2as$

where v = velocity of a body at a certain instant; u = initial velocity; t = time

Example (AMIE S05, 6 marks)

Derive the following equations of motion of a body moving in a straight line with uniform acceleration:

(i) $S = ut + \frac{1}{2}at^2$

(ii) $V^2 = u^2 + 2aS$

Solution

Let "u" is initial velocity and "a "is constant acceleration.

We are interested to find the velocity v and the distance "s' travelled after any time "t".

Acceleration $a = \frac{d^2 s}{dt^2}$

Integrating with respect to t

$$\frac{ds}{dt} = at + c_1$$

where c_1 is a constant of integration.

When t = 0, ds/dt = u Hence c_1 = u

or $$\frac{ds}{dt} = at + u$$

i.e. $$v = u + at \quad (1)$$

Integrating again with respect to t

$$s = \frac{at^2}{2} + ut + c_2$$

where c_2 is another constant of integration.

When t = 0, s = 0. Hence c_2 = 0

$$\therefore \quad s = ut + \frac{1}{2}at^2 \quad (2)$$

From (1), we have

$$t = \frac{v-u}{a}$$

Substituting in (2)

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 = \frac{uv - u^2}{a} + \frac{1}{2}\frac{(v-u)^2}{a}$$

$$2as = 2u(v-u) + (v-u)^2 = (v-u)(v+u)$$

$$2as = v^2 - u^2$$

$$\therefore \quad v^2 = u^2 + 2as \quad (3)$$

Example

The brakes of a train reduce its speed from 60 to 20 km/h while it runs 200 m. Assuming that there exists constant retarding force, find (a) how much further the train will run before coming to rest, and (b) how long will it take.

Solution

Given that $u = 60$ km/h, $v = 20$ km/h, $s = 200$ m $= 0.2$ km

Now $v^2 - u^2 = 2as$

i.e. $400 - 3600 = 2 \times a \times 0.2$

i.e. $a = -3200/0.4 = -8000$ km/h^2

(a) For the train to come to rest, $v = 0$

$$v^2 = u^2 + 2as$$

$$0 = 3600 - 2 \times 8000 \times s$$

i.e. $s = 3600/16000 = 0.225$ km $= 225$ m

Further distance moved $= 225 - 200 = 25$ m

(b) $v = at$

$$0 = 20 - 8000t$$

$\therefore$ $t = 20/8000 = 1/400$ hour $= 9$ sec

Example

Two cars start off to race with velocities v_1 and v_2 and travel in a straight line with uniform acceleration a_1 and a_2. If the result to be dead heat, prove that the length of the course is

$$\frac{2(v_1 - v_2)(v_1 a_2 - v_2 a_1)}{(a_2 - a_1)^2}$$

Solution

Let the length of the course be x km. Since the result of the race is a dead heat, the cars reach the destination at the same time. Suppose each car takes time t hours.

For the first car

$$u = v_1,\ a = a_1,\ t = t,\ s = x$$

$\therefore$ $s = ut + \frac{1}{2}at^2$ gives

$$x = v_1 t + \frac{1}{2}a_1 t^2 \qquad (1)$$

For the second car, we have

$$u = v_2,\ a = a_2,\ t = t,\ s = x$$

$\therefore$
$$x = v_2 t + \frac{1}{2}a_2 t^2 \qquad (2)$$

From eqs. (1) and (2), we get

$$v_1 t + \frac{1}{2}a_1 t^2 = v_2 t + \frac{1}{2}a_2 t^2$$

$$\therefore \qquad t\left[\frac{(a_2 - a_2)t}{2} + (v_2 - v_1)\right] = 0$$

Rejecting the solution t = 0, which corresponds to the initial position, we get

$$t = \frac{2(v_1 - v_2)}{(a_2 - a_1)}$$

Substituting in (1), we get

$$x = \frac{2v_1(v_1 - v_2)}{(a_2 - a_1)} + \frac{1}{2}a_1\left\{\frac{2(v_1 - v_2)}{(a_2 - a_1)}\right\}^2$$

After solving we get

$$x = \frac{2(v_1 - v_2)(v_1 a_2 - v_2 a_1)}{(a_2 - a_1)^2}$$

Problem

A particle moving with uniform acceleration in a straight line passes points A, B, C. If AB = BC = d, and if the time from A to B is t_1 and B to C is t_2, prove that acceleration is

$$\frac{2d(t_1 - t_2)}{t_1 t_2 (t_1 + t_2)}.$$

ACCELERATION DUE TO GRAVITY

A body which is free to move entirely under the influence of the attraction of the earth will be subjected to an acceleration directed towards the centre of the earth. This acceleration is called acceleration due to gravity. When a uniform acceleration is assumed, its value is generally taken as 9.81 m/sec^2. It is denoted by g.

Body Moving Vertically Downward

1. $v = u + gt$
2. $s = ut + \frac{1}{2} gt^2$
3. $v^2 = u^2 + 2gs$

Body Projected Vertically Upwards

1. $v = u - gt$

2. $s = ut - \frac{1}{2}gt^2$
3. $v^2 = u^2 - 2gs$

Body Just Dropped

here u = 0

1. $v = gt$
2. $s = \frac{1}{2}gt^2$
3. $v^2 = 2gs$

Example (AMIE S11, 12, 13, 5 marks)

A stone after falling 4 seconds from rest breaks a glass pane and in breaking it losses 25% of its velocity. How far will it fall in the next second ? g = 9.81 m/s^2.

Solution

Velocity acquired in 4 seconds in falling from rest = v = u + gt = 0 + 9.81 x 4 = 39.24 m/sec.

Since the stone loses 25% of its velocity, its velocity after breaking the glass pane = 39.24 x (3/4) = 29.43 m/s.

$\therefore$ Distance by which the stone will fall in the next one second

$$s = ut + \frac{1}{2}gt^2 = (29.43 \text{ x } 1) \text{ x}(\frac{1}{2} \text{x } 9.81 \text{ x } 1^2)$$

$$= 34.335$$ **Answer**

Example

A particle falls from rest and in the last second of its motion it passes 70 m. Find the height from which it fell and the time of its fall. g = 9.81 m/s^2.

Solution

Let t be the total time of fall.

Distance travelled in t seconds

$$s_1 = \frac{1}{2}gt^2$$

Distance travelled in (t-1) seconds

$$s_2 = \frac{1}{2}g(t-1)^2$$

Distance travelled during the last second of its fall

$s_1 - s_2 = \frac{1}{2}g[t^2 - (t-1)^2] = 70$ m

$\therefore \quad \frac{1}{2}$ x 9.8 x (2t-1) = 70

$\therefore \quad t = 7.64$ s

Height from which the particles fell = $s_1 = \frac{1}{2}$ x 9.81 x (7.64) **= 286.1 m**

Example

A particle is dropped from the top of a tower 200 m high and another particle is projected at the same time vertically upwards from the foot of the tower so as to meet the first particle at a height of 50 m. Find the velocity of projection of the second particle.

Solution

Let the particles meet after t seconds. The first particle falls from rest a distance = 200 - 50 = 150 m.

Hence $\quad 150 = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{150x2}{9.81}} = 5.53 \text{ sec} \qquad (1)$$

If u is the required velocity of the second particle, then

$$50 = ut - \frac{1}{2}gt^2 \qquad (2)$$

Adding eqs (1) and (2), we get

200 = ut

$\therefore \quad u = 200/5.53 = 36.17$ m/s

Example

Two balls are projected simultaneously with the same velocity from the top of a tower, one vertically upwards and the other vertically downwards. If they reach the ground in times t_1 and t_2 respectively, show that $\sqrt{t_1 t_2}$ is the time which each will take to reach the ground if simply let drop from the top of the tower.

Solution

Let h = height of the tower and u = velocity of projection

Taking downward direction positive, for the first ball, we have

$u = -u$, $s = h$, $t = t_1$, $a = g$

$$s = ut + \frac{1}{2}gt^2 \quad \text{gives}$$

$$h = ut_1 + \frac{1}{2}gt_1^{\,2} \qquad (1)$$

For the second ball,

$u = u$, $s = h$, $t = t_2$ $a = g$

$$h = ut_2 + \frac{1}{2}gt_2^{\,2} \qquad (2)$$

Eliminating u from Eqs. (1) and (2), we get

$$h = \frac{1}{2}gt_1t_2$$

If t is the time to reach the ground when initial velocity is zero, we get

$$s = \frac{1}{2}gt^2$$

$$\frac{1}{2}gt_1t_2 = \frac{1}{2}gt^2$$

$\therefore$ $t = \sqrt{t_1t_2}$.

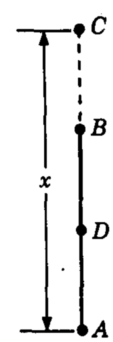
Example

A stone projected vertically upwards from a point A passes a point B after 3 seconds. If it returns to A after a further interval of 4 second, find (i) the height of B above A (ii) the velocity with which the stone passes the point midway between A and B.

Solution

Let u = velocity of projection

x = maximum height reached



Time taken from A to C and back to A

$= 3 + 4 = 7s$

$$0 = ux7 - \frac{1}{2}x9.81x49$$

or $$u = \frac{9.81x49}{2x7} = 34.3 \text{ ms}^{-1}$$

Time taken from A to B = 3s

$$s = ut - \frac{1}{2}gt^2 = 34.3x3 - \frac{1}{2}9.81x9 = 102.9 - 44.1$$

or AB = 58.8 m

$$AD = \frac{1}{2}AB = 29.4m$$

$$v^2 - u^2 = 2gs \Rightarrow v^2 - (34.3)^2 = - 2 \times 9.81 \times 29.4$$

$$v^2 = - 576.83 + 1176.5 = 599.67$$

$\therefore$ $v = 24.5\ ms^{-1}$

Problem

A stone projected vertically upwards from a point O passes a point A three seconds after projection. If it returns to O after a further interval of four seconds, find (a) the height of A above O (b) the velocity with which the stone passes the point midway between O and A.

Answer: 58.86 m, 42.12 m/s

Problem

From a balloon rising vertically upwards with a velocity of 10 m/s a stone is let fall which reaches the ground in 15 seconds. Find how high the balloon was when the stone was stopped.

Answer: 953.625 m

Problem

A particle is projected vertically upwards with a velocity of u m/s and after t seconds another particle is projected upwards from the same point and with the same velocity. Prove that the particles will meet at a height $\frac{4u^2 - g^2t^2}{8g}$ m *after a time* $\left(\frac{t}{2} + \frac{u}{g}\right)$ *seconds from the start.*

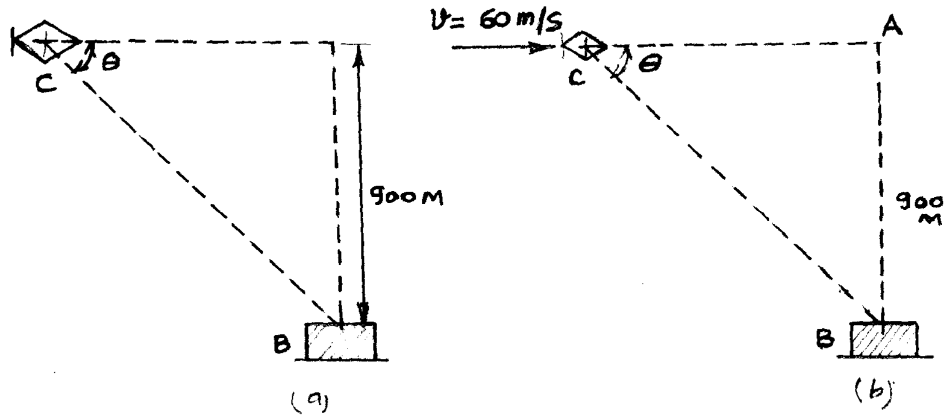
Example(AMIE S 95)

An aircraft carrier C is flying horizontally with a constant speed of 60 m/s at an altitude of 900 m. If the pilot drops a package with the same horizontal speed, determine the angle θ at which he must sight the target B so that the package strikes B. Neglect air resistance.

Solution

Refer to fig (b). This figure shows the given arrangement along with the aircraft carrier speed u equal to 60 m/s.

Let B be the point referred to as the target.



From the point C, the package is dropped with the same horizontal speed u(=60 m/s) so as to strike the target B.

Considering now the vertical component of the package velocity.

At C $\quad u = 0$, and $g = 9.81\ m/s^2$

We know $\quad S = ut + \frac{1}{2}gt^2$

where t = time required by the package to strike the target at B.

Then $\quad 900 = \frac{1}{2} \times 9.81 \times t^2$

$\therefore \quad t = 13.545$ sec.

Horizontal distance CA covered by the package is given by

$$S = u.t = 60 \times 13.545 = 812.7 \text{ m}$$

i.e. the package was released from the aircraft carrier when the horizontal distance is 812.7 m from B.

From fig (b), we see

$$\tan\theta = \frac{AB}{AC} = \frac{900}{812.7} = 1.1074$$

$\therefore \quad \theta = \tan^{-1}1.1074 = 47.92^0$ **Answer**

Example

A lift starts from rest and attains a speed of 4.9 m/sec in 2 secs. Find the weight of a 80 kg main in the lift if the lift is (i) moving up (ii) moving down.

Solution

$$a = (V - u)/t = (4.5 - 0)/2 = 2.25\ m/s^2$$

Let R be the upward reaction of the lift. Apparent weight = R

(i) Lift moving up: R – mg = ma, Putting values, we find R = 98.35 kg

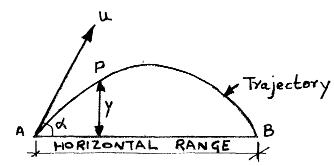
(ii) Lift moving down: mg – R = ma, Hence R = 61.65 kg

Projectiles

Angle of Projection : When a particle is projected into space, the angle between the direction of projection and the horizontal plane through the point of projection is called the angle of projection.

Trajectory : The path traced by a particle projected is called the trajectory of the particle.

Horizontal Range : The distance between the point of projection and the point where the path of the projectile meets the horizontal plane through the point of projection is called the horizontal range.



$$R = u\cos\alpha . T = u\cos\alpha . \frac{2u\sin\alpha}{g} = \frac{u^2}{g}.\sin 2\alpha$$

Range will be maximum when $2\alpha = 90^0$ i.e. $\alpha = 45^0$

$\therefore$ $$R_{max} = \frac{u^2}{g}$$

Time of Flight : This is the duration for which the particle is in motion.

$$T = \frac{2u\sin\alpha}{g}$$

Equation of projectile Path: Projectile path is a parabola whose equation is

$$y = x\tan\alpha - \frac{1}{2}g.\frac{x^2}{u^2\cos^2\alpha}$$

Greatest Height. The greatest height BC = the ordinate of the vertex

$$= \frac{u^2\sin^2\alpha}{2g}$$

Now $$y = u\sin\alpha.t - \frac{1}{2}gt^2$$

At the greatest height, the vertical component of the velocity must vanish. Hence

$$\frac{dy}{dt} = 0$$

or $$u\sin\alpha - gt = 0$$

$$t = \frac{u \sin\alpha}{g}$$

$$\therefore \quad y_{max} = u \sin\alpha . \frac{u \sin\alpha}{g} - \frac{1}{2} g . \frac{u^2 \sin^2\alpha}{g^2}$$

$$= \frac{u^2 \sin^2\alpha}{g} - \frac{1}{2}\frac{u^2 \sin^2\alpha}{g}$$

$$= \frac{u^2 \sin\alpha}{2g}$$

Example

A particle is projected with a velocity of 10 m/s at an angle of elevation 60^0. Find (a) the equation to its path (b) the length of the latus rectum (c) greatest height attained (d) height of the directrix of the path (e) time required for covering the range and (f) length of the range.

Solution

Given u = 10 m/s and $\alpha = 60^0$

(a) The equation to the path is

$$y = x \tan\alpha - \frac{gx^2}{2u^2 \cos^2\alpha} = x \tan 60^0 - \frac{9.81x^2}{2x100x \cos^2 60^0}$$

$$\therefore \quad y = 1.732x - 0.1962x^2$$

(b) Latus rectum = $\frac{2u^2 \cos^2\alpha}{g} = \frac{2x100x(1/4)}{9.81} = 5.097 \text{ m}$

(c) Greatest height = $\frac{u^2 \sin^2\alpha}{2g} = \frac{100x(3/4)}{2x9.81} = 3.823 \text{ m}$

(d) Height of directrix = $\frac{u^2}{2g} = \frac{100}{2x9.81} = 5.097 \text{ m}$

(e) Time of covering the range

$$T = \frac{2u \sin\alpha}{g} = \frac{2x10}{9.81} x \frac{\sqrt{3}}{2} = 1.765 \text{ s}$$

(f) Horizontal range

$$R = \frac{u^2}{g} \sin 2\alpha = \frac{100}{9.81} x \frac{\sqrt{3}}{2} = 8.828 \text{ m}$$

Example

A particle is projected at a velocity u. Find the angle of projection in order the horizontal range is a maximum.

Solution

Let α be the angle of projection.

∴ Range = $\frac{u^2}{g}.\sin 2\alpha$

The range is maximum when sin2α = 1.

i.e. $2\alpha = 90^0$

∴ $\alpha = 45^0$

The angle of projection should be 45^0.

Maximum range = $\frac{u^2}{g}$ **Answer**

Example

A projectile is projected with a velocity of 45 m/s at an angle of elevation of 30^0 with the horizontal . Find,

(i) *The maximum height reached by the projectile*

(ii) *The time of flight*

(iii) *The range of the projectile.*

Solution

u = 45 m/s, $\alpha = 30^0$

Maximum height reached by the projectile = $\frac{u^2\sin^2\alpha}{2g} = \frac{45^2\sin^2 30}{2 \text{ x } 9.81}$ = 25.80 m

Time of flight = $\frac{2u\sin\alpha}{g} = \frac{2 \text{ x } 45\sin 30^0}{9.81}$ = 4.59 sec.

Range of the projectile = $\frac{u^2}{g}.\sin 2\alpha = \frac{45^2}{9.81}.\sin 60^0$ = 178.77 m.

Problem

The maximum range of a projectile is found to be 3000 m. Find the angle of projection for which the range will be 2250 m if the velocity of projection remains the same.

Answer : $\alpha = 24^0 18' \text{ or } 65^0 42'$

Problem

Find the maximum horizontal range of a cricket ball projected with a velocity of 15 m/s.

If the ball is to have a range of 20 m, find the least angle of projection and the least time taken.

Answer: 22.935 m, 30.346^0, 1.545 s

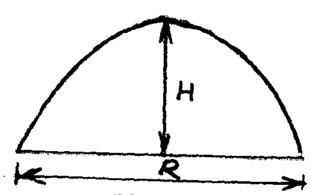
Example(AMIE W 94)

The particles projected at such an angle with the horizontal that the horizontal range is four times the greatest height attained by the particles. Find the angle of projection.

Solution

$$R = \frac{u^2 \sin\alpha}{g} \quad (1)$$

$$H = \frac{u^2 \sin^2\alpha}{2g} \quad (2)$$



Given, H = 4R

Solving (1) and (2)

$\alpha = 86.42^0$ **Answer**

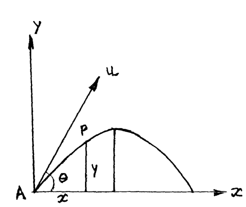
Example(AMIE W 93)

A projectile is fired with initial velocity u at an angle θ with the horizontal from a point A. Taking A as the origin of co ordinate axes x, y; x along the horizontal and y along the vertical, derive the expression of the path of the projectile

$$y = x\tan\theta - \frac{gx^2}{2u^2} \cdot \sec^2\theta$$

Solution

Refer to figure.



Let P be the position of the projectile after t seconds: with Ax and Ay as coordinate axes. Let the coordinates of P be (x,y). Then x is the horizontal distance travelled in t seconds and y the vertical distance.

The initial horizontal velocity is $u\cos\theta$.

$\therefore \quad x = t \times u\cos\theta \quad (1)$

The initial vertical velocity is $u\sin\theta$ with vertical acceleration -g.

$\therefore \quad S = (u\sin\theta)t - \frac{1}{2}.gt^2 \quad (2)$

From (1) and (2)

$$y = u\sin\theta.\frac{x}{u\cos\theta} - \frac{1}{2}.g.\frac{x^2}{u^2\cos^2\theta} = x\tan\theta - \frac{gx^2}{2u^2}.\sec^2\theta \quad (3)$$

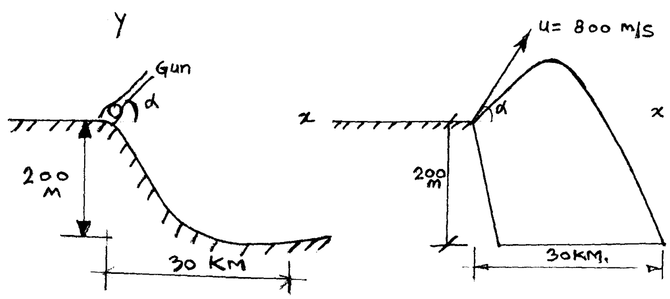
This is the required equation.

Example(AMIE S 93)

A gun is placed on a cliff as shown in the Figure. The muzzle velocity is 800 m/s. At what angle α must the gun point in order to hit a target A as shown in the Figure.

Solution

Refer to figure(b) .



Given u = 800 m/s

coordinates of target A : x = 30,000 m and y = - 200 m

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\alpha}$$

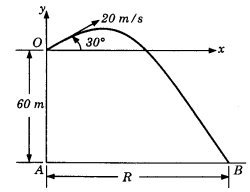
$$200 = 30{,}000 \tan\alpha - \frac{9.81 \times (30{,}000)^2}{2 \times 800^2\cos^2\alpha}$$

solving $\quad 68.98 \tan^2\alpha - 300 \tan\alpha + 66.98 = 0$

$\alpha = 77^0$ and 13^0 **Answer**

Problem

A ball is thrown from the top of a tower 60 m high with a velocity of 20 m/s at an elevation of 30^0 above the horizontal. Find the horizontal distance from the foot of the tower to the point where it hits the ground. ($g = 10$ m/s^2).



Answer: 79.67 m

Problem

A stone is thrown horizontally with a velocity 20 m/s from top of a tower of height 50 m. Find where and with what velocity it will strike the level ground, through the foot of the tower.

Answer: $20\sqrt{10}$ m, $v = 10\sqrt{14}$ m/s, $\theta = \tan^{-1}[(\sqrt{10})/2]$

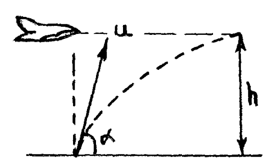
Problem

If R be the range of a projectile on a horizontal plane and h its maximum height, show that the maximum horizontal range with the same velocity of projection is $2h + \dfrac{R^2}{8h}$.

Hint : $R = \dfrac{u^2}{g}.\sin 2\alpha = \dfrac{u^2}{g}\sin\alpha.\cos\alpha$, $h = \dfrac{u^2\sin^2\alpha}{2g}$, $R_{max} = \dfrac{u^2}{g}$

Example

An aero plane is flying at a height h above a level plane with a velocity v. A bullet is fired from a gun at the instant the aero plane is vertically above the gun. Find the angle α of projection of the bullet and also its minimum initial velocity, so that it can hit the aero plane.



Solution

u = velocity of projection of the bullet

Obviously $u\cos\alpha = v$ (i)

and $\dfrac{u^2\sin^2\alpha}{2g}$ = at least h (ii)

$$\sin^2\alpha = \frac{2gh}{u^2} \text{ and } \cos^2\alpha = \frac{v^2}{u^2}$$

$\therefore$ $$\sin^2\alpha + \cos^2\alpha = \frac{2gh}{u^2} + \frac{v^2}{u^2} = 1$$

$\therefore$ $$u = \sqrt{v^2 + 2gh}$$

Newton's Law of Motion, Work, Power, Energy, Impulse, Momentum

FIRST LAW

A body remains in a state of rest or of uniform motion in a straight line, unless it is acted upon by external forces.

SECOND LAW

Rate of change of momentum is proportional to the impressed force and takes place in the direction in which the force acts.

The second law states that the rate of change of momentum is proportional to the resultant force on the body.

If F is the resultant force on the body,

$$F \propto ma$$

or $$F = \lambda ma$$

where λ = constant, a = acceleration

To find the magnitude of the force we have to first select a unit of force.

$$F = \lambda ma$$

if m =1, a = 1, then $\lambda = 1$

Hence if the force acting on a unit mass producing unit acceleration, is taken as the unit of force then $\lambda = 1$.

$\therefore$ $$F = ma$$

The unit of force in S.I. units is Newton. This is the force acting on a mass of 1 kilogram producing an acceleration of 1 m/s^2.

Example

A force of 10 N acts on a body having a mass of 2 kg. for 15 seconds. If the body starts from rest, find the acceleration produced, and the distance moved in this interval.

Solution

Force = mass x acceleration

$\therefore$ $$\text{acceleration } a = \frac{\text{force}}{\text{mass}} = 10/2 = 5 \text{ m/sec}^2$$

Distance moved

$$S = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2 = \frac{1}{2} \text{ x } 5 \text{ x}(15)^2 = 562.5 \text{ metres.}$$

Example

A railway car weighing 60 kN is capable of exerting a tractive effort of 2500 N. Find the acceleration obtained on a level track if the tractive resistance is 6 N per kN of the car's weight.

Solution

Tractive effort = 2500 N

Tractive resistance = 6 x 60 = 360 N

Net driving force = 2500 - 360 = 2140 N

Mass of the car

$$m = W/g = 60{,}000/9.81 \text{ kg.}$$

Let the acceleration be "a" m/sec^2

Force = mass x acceleration

$$2140 = \frac{60{,}000}{9.81}.a$$

$$\therefore \quad a = 0.35 \text{ m/sec}^2$$

Work

If a force (F) acts on a body and causes it to move some distance (s), work (W) is said to be done by that force. Its unit is N-m.

$$W = F \text{ x } s$$

Energy

The capacity to do work is called energy (E). It is measured in N-m.

Kinetic Energy

This is the energy which a body possess because of its velocity.

$$E = \frac{1}{2}mv^2$$

Potential Energy

This is the energy which a body possesses due to its position.

$$E = mgh = Wh \qquad [m = W/g]$$

Relation between Work and Energy

$$W = \Delta E$$

i.e. Work = change in kinetic energy

Work done by Torque or moment

When the body rotates through angle θ, the total work done is

$$W = M\theta$$

Kinetic Energy for Rotation

$$E = \frac{1}{2} I_0 \omega^2$$

where I_0 is mass moment of inertia with respect to axis of rotation.

Kinetic Energy for an object having both Linear and Rotational Motion

Let an object is moving with linear velocity (v) and rotational velocity (ω) then

$$E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

Power

Power $P = W/t$ = Work/time

Also $P = \frac{2\pi NT}{60x1000} = \frac{T\omega}{1000} kW$

where T is the torque in N-m, N the revolutions per minute(rpm), v is velocity in m/s and F is force in N.

1 H.P. = 0.746 kW

Example

A bullet moving at the rate of 60 m/s is fired into a thick target which it penetrates to the extent of 15 cm; if fired into a target 7.5 cm thick with equal velocity, with what velocity would it emerge supposing the resistance to be uniform and the same in both cases.

Solution

Let mass of bullet be m kg. Force of resistance is F N.

Then $W = \Delta E_k$

$$F \, x \, 0.15 = \frac{1}{2} mx(60)^2$$

i.e. F = 12000 mN

Let v be the velocity of emergence then

$$12000m \, x \, 0.075 = \frac{1}{2} m[(60)^2 - v^2]$$

Solving v = 42.426 m/s

Problem

A machine gun bullet weighing 3 x 10^{-3} N is fired with a velocity of 500 m/s. What is the kinetic energy of the bullet? If the bullet can penetrate 30 cm in a block of wood, what is the average resistance of the wood? What will be the exit velocity of the bullet if fired into a similar block of wood 15 cm thick?

Answer: F = 12742 N, v = 353.5 m/s

Impulse

When a force is constant in magnitude and direction, the impulse is the product of the force and the time during which it acts.

$$\text{Impulse} = F \times t$$

Momentum

Momentum is the product of the force and velocity.

$$\text{Momentum} = m \times v$$

Both Impulse and momentum are vector quantities and their units are N-s.

Conservation of Linear Momentum

If the two bodies have masses m_1 and m_2 with velocities v_1 and v_2 respectively, before coming into contact with each other, and velocities v_1' and v_2' at the end of the period of contact. Then according to the conservation of linear momentum, we have

Total momentum (before) = Total Momentum (After)

Example

A gun weighing 80 kN shoots a 250 N projectile in a horizontal direction with a muzzle velocity of 500 m/s. The bullet weights 150 N and is assumed to move with one-half of the muzzle velocity of the projectile. Find the initial recoil velocity of the gun. If the recoil is resisted by a constant force, find this force if the recoil distance is 80 cm.

Solution

Let v be the recoil velocity in m/s.

Now momentum before

$$= \frac{8 \text{x} 10^4}{9.81} v$$

Momentum after

$$= \frac{250}{9.81} \text{x} 500 + \frac{150}{9.81} \text{x} \frac{500}{2}$$

Equating we get

$$8 \times 10^4 v = 125000 + 37500$$

Solving $v = 2.03$ m/s

The gun moves in a direction opposite to that of the projectile.

Using the work energy relation, W = ΔE

$$F \times 0.8 = \frac{1}{2}\frac{80 \times 10^4}{9.81}\left(0 - 2.03^2\right)$$

i.e. F = 21 kN

Example

A body of mass 6 kg moving with a velocity of 3 m/s meets a body of mass 4 kg moving (a) in same direction (b) in opposite direction with a velocity of 1.5 m/s. If they coalesce(combine) into one body, find the velocity of the compound body. Find also the loss of kinetic energy.

Solution

(a) Let v' be the velocity after coalescing(combining).

Momentum before combining

= 6 x 3 + 4 x 1.5

Momentum after combining

= (6 + 4)v'

After Equating

We get v' = 2.4 m/s

Now Kinetic energy before combining

$$= \frac{1}{2} \times 6 \times 3^2 + \frac{1}{2} \times 4 \times 1.5^2$$

K.E. after combining

$$= \frac{1}{2}(6+4)(2.4)^2$$

Change in K.E.

$$= \frac{1}{2} \times 6 \times 3^2 + \frac{1}{2} \times 4 \times 1.5^2 - \frac{1}{2}(6+4)(2.4)^2 = 2.7 \text{ N-m}$$

(b) Momentum before combining

= 6 x 3 - 4 x 1.5

Momentum after combining

= (6 + 4)v'

Equating $v' = 1.2$ m/s

K.E. before = $\frac{1}{2}x6x3^2 + \frac{1}{2}x4x1.5^2$

K.E. After = $\frac{1}{2}(6+4)(1.2)^2$

Change in K.E. = $\frac{1}{2}x6x3^2 - \frac{1}{2}x4x1.5^2 - \frac{1}{2}(6+4)(1.2)^2$ = 24.3 N-m.

Example

A shot of mass m penetrates a thickness s of a fixed plate of mass M. Prove that if M is free to move, the thickness it penetrated is $\frac{s}{\left[1+\frac{m}{M}\right]}$

Solution

Let v be the velocity of the shot and F be the average resistance to penetration. When the plate is fixed, using $W_k = \Delta E_k$, we get

$$F_s = \frac{1}{2}mv^2$$

Let s' be the thickness penetrated when the plate is free to move, then using the principle of conservation of momentum, we have

$$mv = (m + M)v'$$

and using the principle of work and energy, we have

$$Fs' = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)v'^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{m+M}$$

$$= \frac{1}{2}mv^2\left[1 - \frac{m}{m+M}\right] = \frac{1}{2}mv^2\left[\frac{m}{m+M}\right]$$

$$\therefore \qquad s' = \left[\frac{M}{m+M}\right]s = \frac{s}{\left[1-\frac{m}{M}\right]}$$

Problem

A gun weighing 10 kN has a recoil of 1 m when firing a 100 N projectile horizontally. If the recoil is resisted by a nest of springs that is deformed 1 mm by a force of 100 N. Find the initial recoil velocity of the gun and the muzzle velocity of the projectile.

Ans. 9.9 m/s 990 m/s

Problem

A 600 N man running at 5 m/s jumps into a 1 kN boat. The boat is originally moving toward the man at a velocity of 1.5 m/s. Find the velocity of the boat after the man has jumped into it.

Ans. 2.81 m/s

Problem

A bullet of 125 gram strikes a target with a velocity of 400 m/s and is embedded in it. The target mass is 10 kg and is free to move. find the velocity of target after impact and the loss of kinetic energy.

Ans. 4.93 m/s, 9876.5 Nm

COLLISION OF ELASTIC BODIES

If two bodies suddenly collide, an impulsive force, or impact, is set up between them. The sum of the momenta before impact is equal to the sum of the momenta after impact, i.e., the conservation of momentum holds, thus for direct central impact, we have

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Coefficient of Restitution

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

in which the proper sign of the four velocities must be included.

The value of e lies between zero and one. It is zero for perfectly inelastic bodies and one for perfectly elastic bodies.

Direct Impact of Two Spheres

Consider the collision of two spheres, as shown in Fig. 13.5. Then

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad (a)$$

Also $$e = \frac{v_2' - v_1'}{v_1 - v_2} \quad (b)$$

or $$v_2' - v_1' = (v_1 - v_2)$$

$$m_1v_2' - m_1v_1' = m_1e(v_1-v_2) \quad (c)$$

Adding Eqs. (a) and (c), we get

$$(m_1 + m_2)v_2' = m_1v_1 = m_2v_2 + m_1e(v_1 - v_2)$$

$$\therefore \quad v_2' = \frac{m_1v_1(1+e)+v_2(m_2-m_1e)}{m_1+m_2}$$

Similarly, $$v_1' = \frac{v_1(m_1 - m_2 e) + m_2 v_2(1+e)}{m_1 + m_2}$$

If $m_1 = m_2$, then

$$v_1' = \frac{v_1(1-e) + v_2(1+e)}{2}$$

$$v_2' = \frac{v_1(1+e) + v_2(1-e)}{2}$$

Example

A ball of mass 4 kg moving with a velocity of 1.5 m/s is overtaken by a ball of mass 6 kg moving with a velocity of 3 m/s. (a) in the same direction as the first (b) in the opposite direction. If e = 1/5, find the velocity of the two balls after impact. Find also the loss of energy in the first case.

Solution

Given m_1 = 4 kg, v_1 = 1.5 m/s, m_2 = 6 kg, v_2 = 3 m/s, e = 1/5 = 0.2

(a) $$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

i.e. $4 \times 1.5 + 6 \times 3 = 4v_1' + 6v_2'$

$\therefore$ $v_1' + 1.5v_2' = 6$ (1)

Also $$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

i.e. $$0.2 = \frac{v_2' - v_1'}{1.5 - 3}$$

$\therefore$ $v_1' - v_2' = 0.75$ (2)

Solving (1) and (2), we get

v_1' = 2.85 m/s and v_2' = 2.1 m/s

Loss in kinetic energy

$$\Delta E = \frac{1}{2}(m_1v_1^2 + m_2v_2^2) - \frac{1}{2}(m_1v_1'^2 + m_2v_2'^2)$$

$$= \frac{1}{2}(4x2.25 + 6x9) - \frac{1}{2}(4x4.41 + 6x8.1225)$$

= - 1.6875 Nm

(b) $$m_1v_1 - m_2v_2 = m_1v_1' + m_2v_2'$$

i.e. $4 \times 1.5 - 6 \times 3 = 4v_1' + 6v_2'$

$\therefore$ $v_1' + 1.5v_2' = -3$ (1)

Also $0.2 = \dfrac{v_2' - v_1'}{v_1 + v_2} = \dfrac{v_2' - v_1'}{1.5 + 3}$

$\therefore$ $v_2' - v_1' = 0.9$ (2)

Solving, we get

$v_1' = -1.74$ m/s and $v_2' = -0.84$ m/s

(minus sign shows that velocity direction is opposite to their direction before impact)

Problem

A 100 N body moving toward the right with a velocity of 12 m/s collides with a 50 N body moving on the same line at 10 m/s towards the left. Determine the final velocity of each body and the percentage loss of kinetic energy by taking e = 0.8.

Answer: 10.8 m/s, 12.4 m/s

Example

A bullet of mass m, moving with a horizontal velocity v, strike a stationary bock of mass M, suspended by a string of length L. The bullet gets embedded in the block . What is the maximum angle made by the string with the vertical ?

Solution

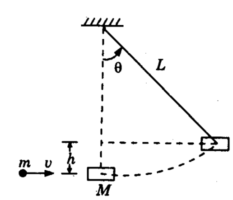
Let v be the velocity of mass (M + m), then, applying law of conservation of momentum,

$$mv = (M + m)V$$

$$V = \frac{mv}{M+m} \quad \text{(i)}$$

Now apply law of conservation of energy,

$$\frac{1}{2}(M+m)V^2 = (M+m)gh$$



$$\frac{1}{2}(M+m)\frac{m^2v^2}{(M+m)^2} = (M+m)gL(1-\cos\theta)$$

or $$\frac{1}{2}\frac{m^2v^2}{M+m} = (M+m)gL(1-\cos\theta)$$

$$\cos\theta = 1 - \frac{m^2v^2}{2gL(M+m)^2}$$

or $$\theta = \cos^{-1}\left[1 - \frac{m^2v^2}{2gL(M+m)^2}\right].$$

Problem

A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after collision the direction of each ball makes an angle of 30^0 with the original line of motion. Find the speed of two balls after the collision. Is kinetic energy conserved in this collision process?

Answer: $3\sqrt{3}$ m/s

Simple Harmonic Motion

The motion of a point is defined as simple harmonic motion when the point moves in a straight line and has an acceleration that is opposite and is directly proportional to its displacement from fixed reference at the midpoint of the path. Consider a point P moving along the path AOB, as shown is Fig. such that the acceleration of P is always equal in magnitude to kx and is always directed towards O, the motion of P will be simple harmonic. This can be expressed as:

$$a = \frac{d^2x}{dt^2} = -kx$$

in which k is a constant and x is the displacement from O.

Now let us consider the motion of a point P moving in a circle as shown in Fig., with a constant angular speed ω. When the point P moves along the circle ABCD, the point Q moves along the horizontal diameter of the circle AC. Let the point P be at the position as shown in the figure after a time t, so that $\theta = \omega t$. Let OQ = x. Then

$$x = \cos\theta = r\cos\omega t$$

$$\therefore \quad \frac{dx}{dt} = v = -r\omega\sin\omega t$$

$$\frac{d^2x}{dt^2} = a = -r\omega^2\cos\omega t = -x\omega^2$$

Now, we shall define some of the terms related to simple harmonic motion.

Amplitude

It is the maximum displacement of the point executing simple harmonic motion from the mean position .

Period (T)

It is time required for one complete oscillation, i.e., the total time taken by point Q in above in traversing the path from O to A, A to C and C to O. thus

$$T = \frac{2\pi}{\omega}$$

Frequency (f)

It is the number of oscillations performed per second. Thus

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Example

The crank of an engine is 22.5 cm and is rotating at 150 rpm. The connecting rod is 90 cm in length. Assuming the motion to be simple harmonic, determine the maximum velocity and acceleration of the piston.

Solution

$$\omega = \frac{2\pi N}{60} = \frac{2\pi x150}{60} = 15.7 \text{ rad/s}$$

Maximum velocity,

$$v_{max} = rw = 0.225 \text{ x } 15.7 = 3.53 \text{ m/s}$$

Maximum acceleration,

$$a_{max} = rw^2 = 0.225 \text{ x } (15.7)^2 = 55.51 \text{ m/s}^2$$

v_{max} occurs at $\theta = 90^0$ or 270^0 and a_{max} occurs when $\theta = 0^0$ or 180^0.

FREE VIBRATIONS

Vibration

To and fro motion of a body about a mean position is called vibration.

Free Vibration

If the system, after it is disturbed, is left to vibrate on its own, the ensuing vibration is known as free vibration.

Natural Frequency

The frequency with which it vibrates of its own is called natural frequency.

Spring

It is device which can store some energy and release it later on.

Stiffness

It is the load per unit deflection.

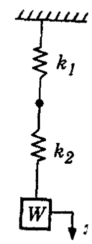
EQUIVALENT SPRINGS

Springs in Series

For the two springs in series having stiffness k_1 and k_2 as shown in figure, we have

Total deflection

$$x = x_1 + x_2$$

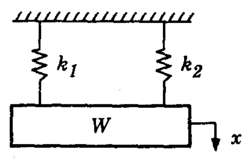


$$\therefore \quad \frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\therefore \quad \frac{1}{k_1} = \frac{1}{k_1} + \frac{1}{k_2}$$

Springs in Parallel

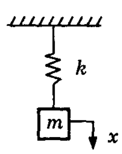
For the two springs in parallel as shown in given figure, we have



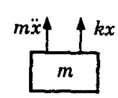
$$W = k_1x + k_2x = kx$$

$$k = k_1 + k_2$$

Consider a single degree of freedom spring-mass system as shown in figure (a). Let the mass m be given a small displacement x in the downward direction. The forces acting on the mass are shown in free body diagram for the mass in figure (b).



(a) Spring-mass system



(b) Free body diagram for mass

These forces are

(a) Inertia force $m\ddot{x}$ acting in the upward direction.

(b) Spring force kx acting upwards

Now $\quad m\ddot{x} + kx = 0$

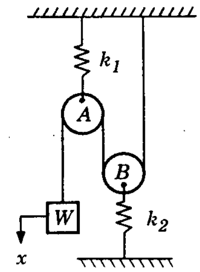
i.e. $\quad \ddot{x} + \frac{k}{m}x = 0$

Let $k/m = \omega_n^2$, then we have

$$\ddot{x} + \omega_n^2 x = 0$$

Example

Determine the frequency of natural vibration of the system shown in given figure.



Solution

Downward displacement of pulley A

$$= \frac{2W}{k_1}$$

Upward displacement of pulley B

$$= \frac{2W}{k_2}$$

Total displacement of weight W = $2\left(\frac{2W}{k_1} + \frac{2W}{k_2}\right)$

Let k_e be the equivalent spring stiffness, then

$$\frac{W}{k_e} = 4W\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$\therefore$ $$\frac{1}{k_e} = 4\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$k_e = \frac{k_1 k_2}{4(k_1 + k_2)}$$

Equation of motion of weight W is therefore

$$\frac{W}{g}\ddot{x} + k_e x = 0$$

or $$\ddot{x} + \frac{k_e g}{W} x = 0$$

$\therefore$ $$\omega_n = \sqrt{\frac{k_e g}{W}} = \sqrt{\frac{k_1 k_2 g}{4(k_1 + k_2)W}}$$

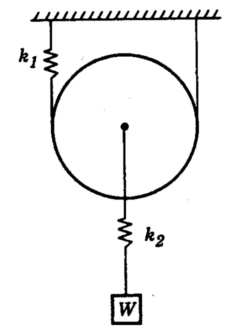
$$\therefore \qquad f_n = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2 g}{4(k_1+k_2)W}}$$

Example

Find the natural frequency of the system shown in figure.

Solution

Neglecting the weight of the pulley, if x is the total deflection, then deflection of the lower spring



$$= \frac{W}{k_2}$$

and deflection of the upper spring

$$= \frac{W}{2k_1}$$

Deflection of pulley

$$= \frac{W}{4k_2}$$

$$\therefore \qquad x = \frac{W}{k_2} + \frac{W}{4k_1} + \frac{W}{k_e}$$

$$k_e = \frac{4k_1k_2}{k_2+4k_1}$$

Equation of motion is : $m\ddot{x} + k_e x = 0$

$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{4k_1k_2g}{4(k_1+k_2)W}}$$

Example

Determine the periods of vibration of weight P attached to spring of stiffness k_1 and k_2 two different cases, as shown in Figure.

Solution

In the case (a) the two springs are in series. Hence

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

$$\omega_n = \sqrt{\frac{k_1 k_2 g}{(k_1 + k_2)P}}$$

$$T = 2\pi\sqrt{\frac{P}{g}.\left(\frac{k_1 + k_2}{k_1 k_2}\right)}$$

In the case (b) the two springs are in parallel. Hence

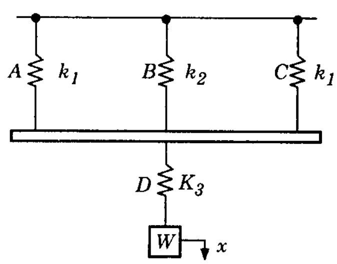
$$k_e = k_1 + k_2$$

$$\omega_n = \sqrt{\frac{(k_1 + k_2)P}{P}}$$

$$T = 2\pi\sqrt{\frac{P}{g}.\frac{1}{(k_1 + k_2)}}$$

Example

The weight W is suspended from a system of springs shown in figure. The rigid bar to which the springs are attached is assumed weightless. If the weight W is pulled down and released, find the frequency and period of its vibrations.



Solution

Springs A, B and C are in parallel, whose equivalent stiffness = $2k_1 + k_2$.

Now spring D is in series with these springs. Hence total equivalent stiffness,

$$\frac{1}{k_e} = \frac{1}{2k_1 + k_2} + \frac{1}{k_3}$$

$$\therefore \qquad k_e = \frac{(2k_1 + k_2)k_3}{2k_1 + k_2 + k_3}$$

Equation of motion is

$$m\ddot{x} + k_e x = 0$$

$$\therefore \qquad \omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{(2k_1 + k_2)k_3 g}{(2k_1 + k_2 + k_3)W}}$$

$$T = 2\pi\sqrt{\frac{(2k_1 + k_2 + k_3)W}{(2k_1 + k_2)k_3 g}}$$

ASSIGNMENT

MOTION

Q.1. (AMIE S11, 12, 13, 4 marks): Derive an expression for the time taken by a body, projected vertically upwards with a velocity u, to reach a height A.

Answer: t = u/g

Q.2. (AMIE S05, 6 marks): Derive the following equations of motion of a body moving in a straight line with uniform acceleration:

(i) $S = ut + \frac{1}{2}at^2$

(ii) $V^2 = u^2 + 2aS$

Q.3. (AMIE S05, 6 marks): A particle starts from rest and moves along a straight line with constant acceleration. If it acquires a velocity of 3.0 metre per second after having travelled a distance of 7.5 metre, find the magnitude of the acceleration.

Answer: 0.6 m/sec^2

Q.4. (AMIE S05, 7 marks): An object is thrown vertically upward with a velocity of 30 m/s. Four seconds later a second object is thrown upward with a velocity of 40 m/s. Determine

(i) the time (after the first object is thrown) when the two objects will meet each other in air

(ii) the height from the earth at which the two objects will meet.

Answer: 2.788 sec, 45.52 m

Q.5. (AMIE W07, 7 marks): A particle moving along a straight line has an acceleration $a = \sqrt{V}$. At time t = 2s, its displacement is 42.67m and velocity is 16m/s. Find the displacement, velocity and acceleration at t = 3 s.

Answer: 56.0025 m, 10.665 m/s, -5.335 m/s^2

Q.6. (AMIE W08, 8 marks): A panicle moves with rectilinear motion and its speed rises from 0 to 12 m/s in 4 s and then drops to 0 in 2 s. Plot the velocity-time curve and find

(i) acceleration in first 4 s and that in last 2 s;

(ii) distance travelled in 6 s; and

(iii) average velocity.

Answer: 3 m/s2, 6 m/s^2; 36 m

Q.7. (AMIE W09, 8 marks): The driver of an automobile, travelling along a straight level highway, suddenly applies the brakes so that the car slides for 2 sec covering a distance of 9.81 m before coming to a stop. Assuming that during this time the car moved with constant deceleration, find the coefficient of friction between the tires and the pavement.

Answer: μ = 0.5,

Q.8. (AMIE S10, 6 marks): A ball is dropped from a height of 100 cm on a smooth floor. The height of the first bounce is 81 cm. Determine the (i) coefficient of restitution, and (ii) expected height of the second bounce.

Answer: 0.9, 0.656 m

Q.9. (AMIE W10, 5 marks): A motorist is travelling at 80 kmph, when he observes a traffic light 200 m ahead of him turns red. The traffic light is timed to stay red for 10 sec. If the motorist wishes to pass the light without stopping, just as it turns green, determine the (i) required uniform deceleration, and (ii) speed as it passes the light.

Answer: -0.444 m/s^2, 17.78 m/s

Q.10. (AMIE S11, 12, 13, 4 marks): A stone after falling from rest for 4 sec breaks a glass plane and in breaking it looses 25% of its velocity. How far will it fall in the next second?

Answer: 34.335 m

Q.11. (AMIE W11, 6 marks): An elevator ascends with an upward acceleration of 1.2 m/s^2. At the instant when the upward speed is 24 m/s, a loose bolt drops from the ceiling of the elevator located 2.75 m from its floor. Calculate the (i) time of flight of the bolt from ceiling to floor of the elevator, (ii) displacement and the distance covered by the bolt during free fall relative to elevator shaft.

Answer: 0.70 sec, 2.31 m, 2.69 m

Q.12. (AMIE W11, 5 marks): A large balloon is rising up with a velocity of 9.81 m/s at an altitude of 39.2 m from the ground. At that instant, a stone of mass 5 kg is dropped from it. After how many seconds will the stone reach the ground?

Answer: 4 sec

Q.13. (AMIE W12, 8 marks): The relation a = 2t defines the motion of a particle. It is given that s = 1.2m and v = 0.6 m/s when t = 1 sec. Find s and v at t = 4 sec.

Answer: 21 m, 15.6 m/s

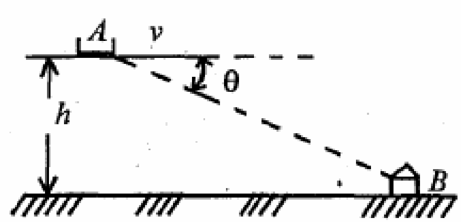
PROJECTILES

Q.14. (AMIE S08, 6 marks): A projectile is fired with an angle of projection a. Find the expression for the trajectory of the projectile. Also, find the maximum range and maximum height of the flight of the projectile.

Q.15. (AMIE S05, 6 marks): Find the initial velocity v_0 with which a projectile would have to be thrown out at the surface of the earth so that it rises to infinite height. Neglect air resistance, and take the radius of the earth 6336 km.

Answer: 7.88 km/sec

Q.16. (AMIE W09, 6 marks): The pilot of an aeroplane, flying horizontally with constant speed K = 480 kmph at an elevation h = 610 m above a level plane, wishes to bomb a target B on the ground. At what angle, θ, below the horizontal should he see the target at the instant of releasing the bomb in order to score a hit ? Neglect air resistance.



Answer: 22.3^0

Q.17. (AMIE S10, 8 marks): A particle is projected with a velocity of 5 m/s at an elevation of 60° to the horizontal. Find the velocity of projection of another particle thrown at an angle of 45° which will have equal (i) horizontal range, (ii) maximum height, and (iii) time of flight.

Answer: 2.207 m, 0.9556 m, 0.8828 m

Now for second particle

4.653 m/s, 6.12 m/s, 6.124 m/s

Q.18. (AMIE W10, 5 marks): A bullet is fired from a height of 120 m at a velocity of 360 kmph at an angle of 30^0 upward. Neglecting air resistance, find (i) total time of flight, (ii) horizontal range of the bullet, (iii) maximum height reached by the bullet, and (iv) final velocity of the bullet just before touching the ground.

Answer: 10.04 sec, 869.46 m, 247.42 m, 86.6 m/s

Q.19. (AMIE S11, 6 marks): A cricket ball thrown from a height of 1.8 m at an angle of 30° from the horizontal with a velocity of 18 m/s is caught by a fieldsman at a height of 60 cm from the ground. How far apart were the two men?

Answer: 15.26 m

Q.20. (AMIE S12, 6 marks): A ball is projected from the ground at an angle of 60 with the horizontal with a velocity of 40 m/s. Find the distance covered by the ball vertically and horizontally after 2 sec.

Answer: 40 m, 49.66 m

Q.21. (AMIE W13, 6 marks): A projectile is aimed at a target on the horizontal plane and falls 12 m short when the angle of projection is 15°, while it overshoots by 24 m when the angle is 45°. Find the angle of projection to hit the target.

Answer: $20^0 54'$

NEWTON'S LAW OF MOTION/WORK/ENERGY/POWER

Q.22. (AMIE S07, 4 marks): State Newton's laws of motion.

Q.23. (AMIE S07, 8 marks): A particle initially at rest is submitted to the action of a force F-kt; where t is the time and k, a constant. Prove that the ratio of displacement x at any time to the velocity of the particle at that time is a linear function of time.

Q.24. (AMIE S08, 8 marks): Two billiard balls of same size and mass collide with the velocities of approach 1.6 m/s and 3.5 m/s, respectively. The angle between their directions of motion is 45°. For a coefficient of restitution of 0.9, what are the final velocities of the balls immediately after the impact? What is the loss in kinetic energy?

Answer: 2.65 m/s and 2.75 m/s

Q.25. (AMIE S10, 6 marks): A bullet, moving at the rate of 300 m/s, is fired into a thick block of target and penetrates up to 0.5 m. If it is fired into a 0.25 m thick target, find the velocity of emergence. The two block are of same material and take the resistance to be uniform in both the cases.

Answer: 212.13 m/sec

Q.26. (AMIE W12, 8 marks): A 150 kg scooterist is travelling with a speed of 36 kmph on a road that makes an angle of 30° with another road upon which a 60 kg cyclist is travelling at 8 kmph. When they approach the crossing, they collide and move as one mass. Determine the final velocity, both in magnitude and direction.

Answer: 27.71 kmph, 2.36^0

Q.27. (AMIE W12, 4 marks): Define force, work, momentum and impulse.

Q.28. (AMIE S11, 6 marks): An engine of mass 50 tonnes pulls a train of mass 250 tonnes up a gradient of 1 in 120 with a uniform speed of 36 kmph. Find the power exerted by the engine, if the tractive resistance is 60 N per tonne.

Answer: 425.25 kW

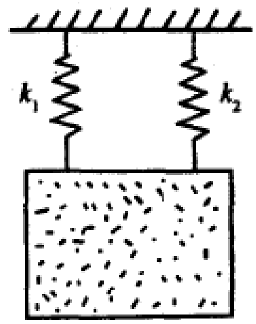
SIMPLE HARMONIC MOTION

Q.29. (AMIE W08, 4 marks): What is simple harmonic motion? Write the equation of motion of a undamped spring-mass system.

Q.30. (AMIE S05, 9 marks): In a mechanism a cross-head moves in a straight guide with simple harmonic motion. At a distance 125 mm and 200mm from its mean position, it has velocities of 6 m/s and 3 m/s respectively. Find the amplitude, maximum velocities and period of vibration. If the cross-head has a mass of 0.2 kg, calculate the maximum force on it in the direction of motion.

Answer: 219.37 mm, 33.3 rad/sec, 0.188 s, 48.6 N

Q.31. (AMIE S08, 6 marks): A vibrating system is shown in Fig. The block is pulled 40 mm down from its equilibrium position and released. For springs, k_1 = 4 kN/m and k_2 = 6 kN/m.



Determine the period of vibration, the minimum velocity of block, and the maximum acceleration of block. The mass of the block is 50 kg.

Answer: 0.444 sec, 0, $\pm$ 8.008 m/s^2

Q.32. (AMIE S09, 8 marks): Explain the principle of free vibration. Derive the equation of motion of a vertically moving undamped spring mass system.

Q.33. (AMIE W11, 3 marks): A small motor of mass 20 kg is symmetrically mounted on four equal springs, each with a spring constant of 25 N/cm. Estimate the frequency and period of vibration of the motor.

Answer: 3.57 cycles/sec, 0.28 sec

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